Silver Grove School

Class-12-A

Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions*

Section A*: Internal choice has been provided in* ***two questions of two marks each, two questions of four marks each and two questions of six marks each.***

Section B: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is **[1]**

a) a symmetric matrix b) a unit matrix

c) a diagonal matrix d) a skew- symmetric matrix

(b) $\int_a^b \frac{\log x}{x} dx$ is equal to **[1]**

a) $\frac{log\ (b-a)}{b-a}$ b) log (a+b).log (b–a)

c) $\log(ab).log(\frac{b}{a})$ d) $\frac{1}{2}\log(ab).log\left(\frac{b}{a}\right)$

(c) The principal value of $\sin^{-1}\frac{1}{2}$ is **[1]**

a) Both $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ b) $\frac{\pi}{6}$

c) $\frac{5\pi}{6}$ d) $\frac{-\pi}{6}$

(d) What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$? **[1]**

a) 6 b) not defined

c) 2 d) 3

(e) A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is **[1]**

a) $\frac{1}{28}$ b) $\frac{3}{28}$

c) $\frac{167}{168}$ d) $\frac{2}{21}$

(f) Number of onto (subjective) functions from A to B if n(A) = 6 and n(B) = 3 are **[1]**

a) 340 b) None of these

c) 2^6 - 2 d) 3^6 - 3

(g) If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$ then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{4x^3}{1-x^4}$ b) $\frac{-4x^3}{1-x^4}$

c) $\frac{1}{4-x^4}$ d) $\frac{-4x}{1-x^4}$

(h) If f(x) = |3 – x| + (3 + x), where (x) denotes the least integer **[1]**

a) neither differentiable nor continuous at x = 3 b) continuous but not differentiable at x = 3

c) differentiable but not continuous at x = 3 d) continuous and differentiable at x = 3

(i) Which of the following is not correct in a given determinant of A, where A = $[a_{ij}]_{3\times3}$. **[1]**

a) Value of a determinant is obtained by multiplying elements of a row or column by corresponding cofactors b) Order of minors and cofactors of elements of A is same

c) Order of minor is less than order of the det (A) d) Minor of an element can never be equal to cofactor of the same element

(j) **Assertion (A):** If A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and C = $\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, then 3A - C = $\begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$. **[1]**

Reason (R): If the matrices A and B are of same order, say m × n, satisfy the commutative law, then A + B = B + A.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

(k) If f and g are real function defined by f (x) = x^2 + 7 and g (x) = 3x + 5, find the f (t) -f (-2) **[1]**

(l) Let A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$. Find A + B **[1]**

(m) Let g be real function defined by and g(x) = $(x + 4)^3$ .Find$\frac{1}{g}$. **[1]**

(n) Let E_1 and E_2 be two independent events such that P(E_1) = p_1 and P(E_2) = p_2. Describe in words of the events whose probability is: $p_1 + p_2 - 2p_1p_2$ **[1]**

(o) A bag contains 5 white, 7 red and 8 black balls. Four balls are drawn one by one with replacement, what is the probability that at least one is white? **[1]**

2. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to x, if x $\in (-\infty, -1)$ **[2]**

OR

Find the points on the curve $y = x^3$ where the slope of the tangent is equal to x-coordinate of the point.

3. Evaluate the Integral: $\int x\sqrt{x^2-1}dx$ **[2]**

4. Prove that function $f(x) = 3x^5 + 40x^3 + 240x$ is increasing on R. **[2]**

5. Evaluate: $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)}dx$ **[2]**

OR

Evaluate: $\int \frac{1}{1+x+x^2+x^3}dx$

6. Let S be the set of all real numbers and let R be a relation in S, defined by $\{R = \{(a,b) : a \leq b^3\}$. Show that R satisfies none of reflexivity, symmetry and transitivity. **[2]**

7. Prove the following. $\cos\left[\sin^{-1}\left(\frac{3}{5}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right] = \frac{6}{5\sqrt{13}}$ **[4]**

8. Evaluate: $\int \frac{x^2}{(x^4-x^2-12)}dx$. **[4]**

9. If $y = (\sin^{-1}x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$ **[4]**

OR

If $y = x^n$ {a cos (log x) + b sin (log x)}, prove that $x^2\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + \left(1+n^2\right)y = 0$

10. **Read the text carefully and answer the questions:** **[4]**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(a) Find the probability that both of them are selected.

(b) The probability that none of them is selected.

(c) Find the probability that only one of them is selected.

(d) Find the probability that atleast one of them is selected.

OR

Read the text carefully and answer the questions: **[4]**

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(a) Calculate the probability that a randomly chosen seed will germinate.

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.

(c) A die is throw and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card.

(d) If A and B are any two events such that P(A) + P(B) - P(A and B) = P(A), then find P(A|B).

11. **Read the text carefully and answer the questions:** **[6]**

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association).

Let A be a 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.

Based on the above information, answer the following questions.

(a) If ₹ 15000 is invested in bond X, then what is the matrix representation of A and B?

(b) If ₹ 15,000 is invested in bond X, how can we determine the total amount of interest received on both bonds?

(c) How much is the investment in two bonds if the trust fund obtains an annual total interest of ₹3200?

12. a. Solve the differential equation for a particular solution: **[6]**

dy = (5x - 4y) dx, when y = 0 and x=0

b. Solve the differential equation $ydx - (x + 2y^2) dy = 0$

OR

Solve the diff. eq $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

13. The sum of the surface areas of a cuboid with sides x, 2x and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal of three times the radius of sphere. Also, find the minimum value of the sum of their volumes. **[6]**

OR

Which of the following functions are decreasing on $(0, \frac{\pi}{2})$?

i. cos x

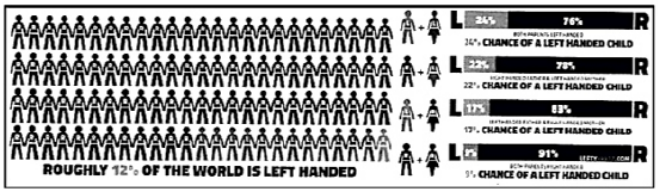
ii. cos 2x

iii. cos 3x

iv. tan x

14. **Read the text carefully and answer the questions:** **[6]**

Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows:

A. When both father and mother are left handed:

Chances of left handed child is 24%.

B. When father is right handed and mother is left handed:

Chances of left handed child is 22%.

C. When father is left handed and mother is right handed:

Chances of left handed child is 17%.

D. When both father and mother are right handed:

Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

(a) Find P $\left(\frac{L}{C}\right)$

(b) Find P $\left(\frac{\bar{L}}{A}\right)$

(c) Find P $\left(\frac{A}{L}\right)$

(d) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) The vectors $3\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\hat{\imath} + \lambda\hat{\jmath} - \hat{k}$ are coplanar if the value of λ is: **[1]**

a) -2 b) 0

c) 2 d) Any real number

(b) If a line makes an angle of 30°, 60°, 90° with the positive direction of x, y, z-axes, respectively, then find its direction cosines. **[1]**

(c) Find the value of $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$. **[1]**

(d) Determine whether the given plane 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z – 1 = 0 are parallel or perpendicular, and in case they are neither, find the angles between them. **[1]**

a) The planes are at 45° b) The planes are parallel

c) The planes are at 55° d) The planes are perpendicular

(e) Find the direction cosines of the line joining the points P(4, 3, -5) and Q(-2, 1, -8). **[1]**

16. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors, then prove that $|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$ **[2]**

OR

If $\vec{a} = (\hat{i} - 2\hat{j}), \vec{b} = (2\hat{i} - 3\hat{j})$ and $\vec{c} = (2\hat{i} + 3\hat{k})$, find $(\vec{a} + \vec{b} + \vec{c})$.

17. Write the value of k for which the planes x - 2 y + kz = 4 and 2x + 5y - z = 9 are perpendicular. **[4]**

OR

Find the angle between the lines whose direction cosines are given by the equations: 3l + m + 5n = 0 and 6mn - 2nl +

5lm = 0.

18. Using integration, find the area of the region bounded by the line y - 1 = x, the x -axis and the ordinates x = - 2 and x = 3. **[4]**